Ján Karabáš
Habilitation thesis
Report by Gareth Jones

Mgr. Ján Karabáš has submitted a habilitation thesis entitled "Discrete Group Actions: Algorithms and Applications". This thesis, of 141 pages, consists of two parts of approximately equal lengths: the first part can be regarded as an introduction to, explanation of, and commentary on the second part, which consists of reprints of four research papers, for all of which the candidate is a co-author. Three of these have been published or accepted by international periodicals with high standards, while the fourth represents work in progress. They lie within the general area of topological graph theory, and they are mainly concerned with the symmetry properties of maps and hypermaps on surfaces (usually, though not always, assumed to be compact and oriented).

In any area of mathematics, the study of symmetry almost invariably uses the concepts of groups and their actions. In this case, the assumed compactness of the structures considered means that the groups involved are all finite, though it is also very useful to regard them as quotients of certain infinite groups, namely Fuchsian groups and in particular triangle groups. There are very well-developed theories of (abstract) finite groups, of finite group actions, and of Fuchsian groups, all of which have been successfully applied to the study of maps and hypermaps. Such applications date back to the development of complex function theory in the late 19th century, but they received an added impetus a century later from Grothendieck's theory of dessins d'enfants, which uses maps and hypermaps to form links between algebraic geometry, Riemann surface theory and the Galois theory of algebraic number fields.

A parallel development, in recent decades, has been the increased use of computers, together with the algorithms embodied in systems such as GAP and MAPLE, to solve problems about finite (or, more generally, finitely presented) groups which are beyond the scope of calculation by hand. This has resulted in a number of enumerations and classifications of various structures of increasing size, leading in turn to a greater understanding of this subject, and in particular an enhanced ability to formulate and test conjectures based on this experimental evidence.

This represents the context in which this thesis has been written. In order to succeed in work of this nature, one must first understand the mathematical background, based on the inter-relationships between areas such as algebra, analysis, combinatorics and geometry. Secondly, one must have sufficient grasp of the strengths and weaknesses of the available computational systems, in order to reformulate theoretical problems into algorithms which are capable of solving them: here efficiency is just as important as accuracy, since there is little practical value in an algorithm which takes a human lifetime to terminate. Finally, one must be able to evaluate the output of the algorithms, and to translate it, in the form of published theorems, enumerations and classifications, into usable new information and understanding in the relevant areas.

The first part of the thesis contains a short introduction, summarising the thesis, followed by a long chapter describing the foundations of topological graph theory and its connections with Riemann surface theory. This is followed by a short chapter on discrete group actions, with an emphasis on algorithmic aspects. Taken together, these two chapters give an impressively clear and concise summary of the subject. There follow two chapters, on symmetry properties and on operations on hypermaps, which give the context for the four papers in the second part of the thesis, and which give commentaries on the methods and results contained in them. There is a detailed bibliography of 102 research papers and monographs, showing evidence of thorough reading of the background and of current developments in the relevant areas. Throughout this part of the thesis, the material is well organised and clearly explained. It is written in good English, though with a few minor errors of syntax or style, and some unimportant typographic errors.

The second part of the thesis consists of four research papers. The first of them, written with Roman Nedela and published in *Electronic Notes in Discrete Mathematics* in 2007, extends the classification of Archimedean (vertex-transitive but non-regular) solids, due to Plato, Archimedes and Kepler, to Archimedean maps of genus 2 (those of genus 0 or 1 are well known). This is done by factoring out the orientation-preserving automorphism group to obtain a one-vertex quotient map; using computers, all such quotient maps are classified, and then their vertex-transitive covers are retrieved by appropriate voltage assignments. The result is a table describing the Archimedean maps of genus 2, giving such useful data as their local vertex type, size, valency, automorphism group, and reflexibility or chirality. In principle, this method can be applied to such maps of higher genus, and indeed this extension is carried out, for genera $g = 2, 3$ and 4, in the second paper, by the same two authors, published in *Mathematics of Computation* in 2012. This paper is a significant achievement, classifying several hundred Archimedean maps, with full details available in an associated website. However, this paper does not cite the earlier one, which is also only briefly mentioned in this thesis; I would have liked to see some comments on the relationship between these two papers, and in particular on the slightly different definitions and classifications given in them for maps of genus 2. It would also be interesting to see drawings of at least some of the Archimedean maps of low genus.

These two authors are joined by Antonio Breda d'Azevedo and Domenico Catalano in a third paper, on maps of Archimedean class and operations on dessins, which has been accepted to appear in *Discrete Mathematics*, a periodical with particularly high standards. In this paper, the category of maps is extended to that of hypermaps (equivalently dessins), and the authors study a set of ten operations, corresponding to ten inclusions between triangle groups, which can be used to convert a regular dessin into a vertex-transitive map on the same surface. This set of operations is shown to be complete, in the sense that any vertex-transitive map obtained from a regular dessin must arise from applying one of these operations.

The fourth and final paper, written by Ján Karabáš and Roman Nedela, is described in this thesis as "in progress", though it has all the appearance of a finished paper. In it, a rather stronger symmetry condition for maps is studied, namely edge-transitivity rather than vertex-transitivity. The authors classify edge-transitive maps into eight classes, according to the quotient by the orientation-preserving automorphism group. This is a rather less refined classification than that used in the monograph by Graver and Watkins, and subsequently by Orbanić, Pellicer, Pisanski and Tucker, where taking the quotient by the full automorphism group gives 14 classes. The advantage of this new approach is that the possible automorphism groups arise as quotients of eight rather straightforward Fuchsian groups, rather than 14 more complicated NEC groups, as in the earlier approach. This is particularly important in using computers to enumerate edge-transitive maps of small genus: whereas Orbanić et al. were able to do this up to genus 4, this more efficient approach allows an enumeration up to genus 10. This is a significant improvement, and the paper explaining it is impressive, though I would have liked to see more details of how the eight classes correspond to the families described by Graver and Watkins.

All four of these papers are written with co-authors. I have known all of them, including the candidate, for a number of years, and I have read their papers, heard their conference talks, and discussed research with them. This knowledge convinces me that the candidate has played a full part in the research described in these papers, and that he can be justified in including them in this thesis.

To summarise briefly, I think that this collection of papers, together with the commentary given in the first part of the thesis, represents a significant contribution to our understanding and knowledge of this particular branch of topological graph theory, and that the ideas described here could form the basis for further interesting research in this area. **I therefore congratulate the candidate on his thesis, and I recommend that it should be approved for the award of the title "doc." (Associate Professor).**

Gareth Jones

Emeritus Professor of Mathematics

University of Southampton

13 March 2015.

Gareth A. Jones